

# DECISION IN THE BIANNUAL PLANNING OF THE FOREST HARVEST: THE USE OF THE MIXED INTEGER PROGRAMMING

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## ABSTRACT

This study was generated in an enterprise with forest areas with eucalyptus, in the North region of Minas Gerais. It presents a mathematical model that supports the decision in the biannual planning of the forest harvest. 204 sites with average productivity of 205 st(steres)/ha were available for cutting. The objective of the proposed model is the optimization of the forest harvest, minimizing the cost of the feller-buncher tractor. The Mixed Integer Programming technique was used to create the model. Of the available sites, the decision support model selected 46 to be harvested in the first year and 42 in the second with a cutting cost of US\$ 91,942.00 and US\$ 79,379.00 respectively, providing the timber demanded by the enterprise that was studied. The results obtained allowed us to verify that the productivity of the sites available for cutting were above 150 st/ha and the decision of not harvesting sites below these levels could represent an economy of up to US\$ 70,000.00 per year.

**Key words:** Optimization, mathematical programming, forest harvest.

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## 1 Introduction

The planning of forest harvest is essential in the coordination of the activities. It allows optimization of the machinery use, regulation of timber flow, productivity increase and costs reduction. For this, a great number of factors must be considered, such as: size and uniformity of the sites; age of the plants; volume increase from one year to another; volume, productivity and estimated costs; distance between the planting area and the log unloading area, among others. These factors must be considered to make it possible to attend to the timber demand of the period in question, optimizing at the maximum the production factors.

Valverde et al. (1996b) conducted a technical analysis of the timber cutting with the feller-buncher forest tractor. They divided the operational cycle in movement without load, cutting, movement between trees and movement for unloading. They used individual time monitoring, observing that the only variable that affected directly the cutting time was the volume of timber per hectare.

In another study, Valverde et al. (1996a) while analyzing the skidder forest tractor, observed that its production capacity (volume of timber extracted per unit of time) was affected by the volume per hectare and by the skidding distance. The highest values, and consequently, the lower costs were obtained in classes of a larger volume per hectare and in the belts of smaller skidding distance.

Linear programming has been used to solve problems in agribusiness and agroforestry systems for a long time. This is particularly due to the complexity of these systems and to the great number of interrelations in the many sub-systems that compose them. According to Davis and Johnson (1987), from the 70's until now the solution of forest planning problems through the use of linear programming has incremented quickly. Since then, a great variety of software has been developed for this purpose, such as: SIMAC, MAX MILLION, RAM, TREES, LCHO, MUSYC and FORPLAN, among others.

Many authors have used this technique for planning the temperate forest cutting with its production mainly destined to the cellulose industry. Ware and Clutter (1971) created a linear programming model to program the harvest in *pinus* industrial forests in the Southeast of the USA, using restricted handiwork availability. This restriction follows the same line of machinery restriction, since limitation of the work force has similar characteristics.

Nelson, Brodie and Sessions (1991), and Jones, Meneghin and Kirby (1991) used Mixed Integer Programming (MIP) in the construction of support models to the short-term planning of the harvest of temperate forests, suggesting clear cut and using timber flow restriction. The Integer Programming technique and its derivations made possible a closer approach of the operational research to cases in which the model's result could not be divided in parts. In the forest planning area, this technique is used to guarantee that, provided that a certain unit is elected to be cut in a determined date, all its area will be used and not only part of it.

This study presents a system of management planning that aims the optimization of the forest harvest. Specifically, a decision model in the planning of the forest harvest that minimizes the costs is considered, simulating situations of optimum allocation of the feller-buncher forest tractor in quick growing forests.

## **2 Methodology**

### **2.1 Study Area and Data Source**

The forest area that was studied belongs to MANESMANN FLORESTAL (MAFLA) enterprise, situated near the city of Bocaiúva in the North of Minas Gerais state. It has a total of 47.577 ha, of which 33.456 ha are planted with eucalyptus. The preliminary forest inventory was used as a reference. It is a preparation for the harvest planning where all the sites available for harvesting and their characteristics of volume of

timber, localization, annual increase, among others, are surveyed. The yield of the feller-buncher forest tractor was obtained after analysis in different productivity levels of the forest. Its fuel consumption, operators' handiwork, maintenance and repair data were surveyed based on the machine's cost control and was converted in dollars.

The timber harvest is done by a group of machines called "harvest modulus". These are responsible for the timber's cutting, extraction, loading and transport to the charcoal making operations. The timber produced is entirely destined to the production of charcoal used as energy source for iron and steelwork. In the case that was studied, there was the timber "harvest modulus" with a totally mechanized system: the cutting was made by the feller-buncher forest tractor, the extraction by the skidder and the sawing by the bucking dog.

## **2.2 Analytical Model and Variables Operating**

According to Machado (1994), forest planning must be elaborated using scientific methodology. It must be supported by logical conditions, based on data collected in real situations and when possible, extrapolated to new situations. The timber flow is directly related to the enterprise's demand and it is a decisive factor in the exploitation's intensity. This makes the dimensioning of all the required forest machines and equipment important, so that the process used in the exploration do not strangle.

In this study the Forest Harvest Planning Support (FHPS) mathematical model was elaborated using Mixed Integer Programming (MIP). MIP is an association of linear programming with integer programming, described as a combination of continuous and discrete variables, in which the integer variables are of the zero-one (0-1) type.

This model was developed to support the elaboration of the forest harvest schedule. This schedule must define which sites, among the ones that are available for cutting, must be harvested and in which sequence. The model works according to inputs such as: sites available for cutting,

machine-hour-cost and field capacity of the feller-buncher forest tractor.

The time interval observed was of two years, which fits in the short-term category. The cutting units are specified for each year. The model that was developed considers the interrelations between the forest areas, characterizing, therefore, the forest management.

Some steps must be followed to adjust all the input and its interrelations. In the first step, the total hour cost of the feller-buncher tractor is defined, which consists of the fixed plus the variable cost per total number of hours.

The second step is the definition of the tractor's yield for different levels of site productivity, that is Field Capacity (FC). This indicator is represented by the area worked by the tractor per time interval and varies according to the volume of timber per forest hectare. The enterprise that was studied gave us the tractor's yield data for nine productivity levels, varying from 60 to 260 steres (st) per hectare.

The third step is to calculate the cutting cost per hectare for different forest productivity levels. This fact is decisive in the selection of the sites that must be cut and is used by the model as coefficient ( $C_{ij}$ ) of the decision variable. Its calculus depends on the machine's hour-cost and on the number of hectares cut in an hour, that is, on the field capacity. Therefore, for each productivity level there will be a coefficient of cutting cost.

The adjustment cost characterizes the integer programming with variables of the 0-1 type and does not depend on the cutting cost. The integer variable is associated to the site that will be selected for cutting, automatically aggregating an integer value to the objective function. Its value may be individual for each site like the machines' transportation cost that depends on the distances, or may be the same value for all the sites.

In the case that was studied, the adjustment cost was considered the same for all the sites with the value of US\$ 100,00. In this case this value has the effect of diminishing the tendency of choosing very small

areas, for its participation in the final cost will be proportionally larger.

The fourth and last step refers to the definition of the decision variables. The enterprise that was studied has 47.577 hectares of planted forests and at each period must dispose a part of the forest to be cut. The disposed forest area is divided in sites with different productivity and consequently with different cutting costs per hectare.

The enterprise disposed 204 sites to be selected for cutting in two years. These sites were divided in different productivity classes that vary from 55 to 255 st/ha, with a class change at every 10 units. For the different classes different cutting costs per hectare are associated. Among the sites that are available for cutting, the ones that can be cut in the first year (July 1999 to June 2000) and in the second year (July 2000 to June 2001) will be selected. This supports, therefore, the planning and elaboration of the planting schedule.

The FHPS mathematical model can be expressed by:

MINIMIZE

$$Z = \sum_{i=1}^I \sum_{j=1}^J (C_{ij} \cdot X_{ij} + CC_{ij} \cdot Y_{ij}) \quad (1)$$

subject to

$$\sum_{i=1}^I \sum_{j=1}^J V_{ij} \cdot X_{ij} = VT_j \quad (2)$$

$$\sum_{i=1}^I \sum_{j=1}^J X_{ij} = A_i \quad (3)$$

$$\sum_{i=1}^I \sum_{j=1}^J X_{ij} - \sum_{i=1}^I \sum_{j=1}^J A_i \cdot Y_{ij} = 0 \quad (4)$$

in which:

$C_{ij}$  = cutting cost of 1 ha of the site  $i$  in moment  $j$ ;

$X_{ij}$  = number of hectares cut in site  $i$  in moment  $j$ ;

$CC_{ij}$  = adjustment cost to initiate cutting in site  $i$  in moment  $j$ ;

$Y_{ij}$  = zero-one type variable associated to the adjustment cost of site  $i$  in moment  $j$ ;

$V_{ij}$  = timber volume in 1 ha of site  $i$  in moment  $j$ ;

$VT_j$  = minimum total volume of timber cut in period  $j$ ;

$A_i$  = site area in hectares.

The objective function refers to the minimization of the costs involved in the forest harvest. This, therefore, supports the planning in the moment of the decision of which sites will be harvested in period  $j$ . Period  $j$  may be any sub-division of the time interval. For this case we considered a year as a sub-division of a 2-year period, that is, all the sites have the possibility of being harvested in year 1 or in year 2. This way, the same site creates two decision variables:  $X_{11}$  and  $X_{12}$ , that is, the number of hectares harvested in site 1 in year 1 and the number of hectares cut in site 1 in year 2. The sites elected for cutting will be clear cut, that is, all the timber from the site will be extracted.

The restriction (2) restricts the total volume that will be harvested as larger or equal to the minimum demanded for the period in question.

The model has as main objective to achieve the volume of timber demanded with the lowest cutting cost. The 204 sites can be chosen in two years to achieve this volume, with different localization and productivity. This allows an infinite number of combinations. The volume restriction assures this objective, making the total timber volume from

the sites the same as the volume demanded for each year.

The site that is selected must be completely cut and the number of hectares that will be cut must not exceed its size. For this, the area restriction limits the number of hectares cut in the selected site to, at the maximum, its own size. This condition is reinforced by the adjustment restriction. This restriction adds the cost of US\$ 100,00 per hectare to the objective function, independent of the number of hectares cut in the site.

Due to the restriction (4), the variable  $Y_{ij}$  will always assume the values 0 or 1. This result occurs because the adjustment restriction must always be smaller than or equal to zero. When the site  $X_{ij}$  is not selected, its value will be null and consequently the variable  $Y_{ij}$  also will be null. While its value is higher than zero,  $X_{ij}$  will be equal to  $A_{ij}$  that is the same coefficient of the variable  $Y_{ij}$ , this demands that variable  $Y_{ij}$  be equal to 1. This is due to the fact that the second member of the equation (4) is null. This restriction demands the variable  $Y_{ij}$  to always present integer values, therefore characterizing the integer programming of the model.

### 3 Results and Discussion

The model was used to try to find the selection of sites to be cut in order to achieve the demanded volume of timber with the smallest total cutting cost possible.

With higher forest productivity levels the cutting cost per volume of timber decreases and consequently the operation of the machine is less expensive. That is, the timber volume is achieved in less operation time and, this way, with a smaller cutting cost. If the variables of the model were simplified to these relations we could simply choose the sites with larger volumes and, as a consequence, we would achieve the lowest cutting cost for the enterprise.

However, the FHPS model considers the possibility of harvesting a forest in two periods. In the first period the volume of timber per hectare (productivity) is the actual volume and in the second period the forest



growth (increase) until cutting time is considered.

In the 204 sites disposed, a total volume of timber of 918.374 st was achieved. For the periods in question, the enterprise demanded 258.000 st of timber in the first year (July 1999 to June 2000) and 270.000 st in the second (July 2000 to June 2001) with the objective of producing enough charcoal to attend the iron and steel work demand.

For the enterprise that was studied, of the 204 available sites 408 variables were created. This is due to the possibility of cutting the sites in two distinct moments (year 1 and year 2). As this simulation considers 2 years, there are 3 possibilities for each variable: to be elected for cutting in the first year, to be elected for the second year or to not be selected at all.

For the first planning year, the model selected 46 of the 204 available sites, mounting a total of 258.000 st. While for the second year it selected 42 sites mounting 270.000 st. This way, the demanded volume of timber for the two periods was achieved.

The minimization of the cutting cost in the elaboration of the harvest plan for year 1 and year 2 can be found in Tables 1 and 2. Table 1 represents the optimum solution for those sites that must be harvested in the first year and Table 2 for the second year. The variable  $X_{ij}$  represents the area of site  $i$  to be cut in the year  $j$ . The number correspondent to index  $i$  is the numeration of the disposed sites. While index  $j$  represents 1 for the first year and 2 for the second.

The timber supply is larger than the demand in two years. This implied that some disposed sites should not be selected for cutting in any of the planning periods, being required to remain growing. On the contrary they should enter a new optimization if any changes are expected in the scenario.

By analyzing the results of the biannual planning made by the FHPS model we constructed Tables 3 and 4 of frequency distribution per productivity levels. These Tables contain the productivity levels (st/ha), the number of sites selected for each productivity level, the total number

of sites disposed per level and the percentile of the sites selected by productivity.

Table 1. Sites selected to compose the cutting schedule of the studied enterprise in year 1 (1999-2000)

Sites	Area (ha)	Increase (st/ha)	Productivity (st/ha)	Class	Cutting Cost (US\$/ha)
X 29 1	37	0	147	145	56
X 31 1	36	0	148	145	56
X 32 1	41	0	149	145	56
X 34 1	25	0	159	155	58
X 35 1	49	0	162	165	61
X 41 1	41	0	179	175	64
X 42 1	14	0	186	185	67
X 43 1	11	0	194	195	68
X 44 1	16	0	205	205	70
X 45 1	42	0	209	205	70
X 46 1	10	0	219	215	73
X 49 1	22	0	269	265	82
X 53 1	46	0	210	205	70
X 57 1	32	0	176	175	64
X 60 1	28	0	199	195	68
X 61 1	24	0	191	195	68
X 63 1	37	0	200	195	68
X 64 1	32	0	152	155	58
X 68 1	24	0	156	155	58
X 69 1	13	0	229	225	75
X 79 1	33	0	217	215	73
X 121 1	26	0	150	145	56
X 124 1	24	0	228	225	75
X 126 1	37	0	205	205	70
X 131 1	38	0	163	165	61
X 132 1	34	0	169	165	61
X 136 1	39	0	157	155	58
X 138 1	38	0	156	155	58
X 140 1	39	0	196	195	68
X 141 1	27	0	157	155	58
X 142 1	38	0	163	165	61

Table 1. Sites selected to compose the cutting schedule of the studied enterprise in year 1 (1999-2000).(continue)

Sites	Area (ha)	Increase (st/ha)	Productivity (st/ha)	Class	Cutting Cost (US\$/ha)
X 145 1	37	0	150	145	56
X 146 1	36	0	150	145	56
X 148 1	38	0	209	205	70
X 153 1	36	0	218	215	73
X 154 1	39	0	220	215	73
X 155 1	34	0	192	195	68
X 164 1	37	0	161	165	61
X 165 1	34	0	170	165	61
X 177 1	18	0	234	235	77
X 182 1	13	0	161	165	61
X 183 1	24	0	199	195	68
X 184 1	18	0	208	205	70
X 192 1	25	0	168	165	61
X 194 1	25	0	185	185	67
X 202 1	24	0	204	205	70

Source: Research Data

Table 2. Sites selected to compose the cutting schedule of the studied enterprise in year 2 (2000-2001)

Sites	Area (ha)	Increase (st/ha)	Productivity (st/ha)	Class	Cutting Cost (US\$/ha)
X 33 2	19	30	184	185	67
X 37 2	5	34	207	205	70
X 38 2	15	34	208	205	70
X 39 2	27	34	208	205	70
X 47 2	27	46	280	275	84
X 48 2	27	51	308	305	87
X 50 2	34	54	328	325	89
X 51 2	6	55	335	335	90
X 52 2	19	59	355	355	91
X 56 2	37	32	194	195	68
X 58 2	35	45	275	275	84
X 62 2	11	44	264	265	82

Table 2. Sites selected to compose the cutting schedule of the studied enterprise in year 2 (2000-2001).(continue)

Sites	Area (ha)	Increase (st/ha)	Productivity (st/ha)	Class	Cutting Cost (US\$/ha)
X 67 2	4	56	336	335	90
X 70 2	36	63	381	385	92
X 71 2	8	65	394	395	92
X 72 2	9	51	308	305	87
X 76 2	8	47	286	285	85
X 77 2	20	70	421	395	92
X 78 2	23	46	277	275	84
X 80 2	15	73	438	395	92
X 81 2	15	48	291	295	86
X 82 2	8	59	357	355	91
X 83 2	34	54	328	325	89
X 84 2	25	50	305	305	87
X 85 2	26	46	281	285	85
X 86 2	21	50	300	295	86
X 87 2	24	60	365	365	91
X 88 2	11	42	256	255	80
X 122 2	37	42	255	255	80
X 123 2	23	44	265	265	82
X 129 2	38	34	209	205	70
X 133 2	38	34	208	205	70
X 139 2	29	42	255	255	80
X 144 2	26	46	276	275	84
X 150 2	25	55	336	335	90
X 151 2	37	42	255	255	80
X 156 2	23	25	150	145	56
X 179 2	20	47	283	285	85
X 180 2	24	47	282	285	85
X 185 2	26	42	254	255	80
X 195 2	25	45	272	275	84
X 201 2	24	44	267	265	82

Source: Research Data

In year 1, a percentage of 84.8% of the selected sites were between the 150 to 225 st/ha levels. As may be observed in Table 3, there is a distribution with a tendency of concentration between the 150-175, 175-200, 200-225 st/ha levels.

An opposite behavior is observed in year 2 (Table 2) when 81% of the selected sites are at productivity levels above 250 st/ha. This distribution may be explained by the increase in the number of sites that enter this level when their timber volume increases. This is due to the fact that they receive an increase in the productivity correspondent to one year's growth and to the reduced number of sites with inferior productivity levels, for most of these were selected in year 1. In both cases the incidence of selecting sites with productivity below 150 st/ha was low, which indicates that below this level the cutting cost is high.

With the selection made by the FHPS model in the first year, if the planner follows the proposal of the sites that should be harvested, the volume of 258.000 st will be achieved with a cutting cost of US\$ 79.379,00. The studied enterprise chooses sites with productivity levels above 100 st/ha as an intuitive parameter for the definition of the viability of using the machine.

To clarify the visualization of the cost reduction with the use of the FHPS model we made a comparison in two scenarios. In the traditional scenario the sites chosen for cutting are between 100 and 150 st/ha. In the optimized scenario, we used as data the result of the model used with sites selected between the 125 and above 250 st/ha levels (Table 5). Its result reflects an economy of about US\$ 70.000,00 per year when the sites proposed by the system are harvested.

The proceeding for biannual planning may be repeated in larger or smaller time intervals, following the same logic of the model. For this we simply make the adjustments in the site repetitions and in the total volume per period. From the data of Tables 1 and 2, which are the results of the interaction of the decision variables of the proposed model, the planner may do the monthly detailing of the sites that will be harvested elaborating the enterprise's annual harvesting schedule.

Table 3. Frequency distribution of the sites selected to be cut in year 1 (1999-2000) by the FHPS model

Productivity Levels (st/ha)	<125	125-150	150-175	175-200	200-225	225-250	>250
Selected Number of Sites	0	3	17	10	12	3	1
Number of Total Sites	81	32	27	12	21	13	18
% of Selected Sites	0	6,5	37	21,8	26,0	6,5	2,2

Source: Research Data

Table 4. Frequency distribution of the sites selected to be cut in year 2 (2000-2001) by the FHPS model

Productivity Levels (st/ha)	<125	125-150	150-175	175-200	200-225	225-250	>250
Selected Number of Sites	0	0	1	2	5	0	34
Number of Total Sites	62	21	24	7	7	1	36
% of Selected Sites	0	0	2,3	4,7	12,0	0	81,0

Source: Research Data

Table 5. Comparison of the optimum and traditional cutting cost for the planning period of 1999-2001

Specification	Year 1 - 258.000 st *	Year 2 - 270.000 st *
Optimized Cutting Cost	91.942,00	79.379,00
Optimized Cost	0,356	0,293
Traditional Cutting Cost (150 st)	163.000,00	149.349,00
Traditional Cost (150 st) Cost Reduction	0,631 71.058,00	0,553 69.970,00

Source: Research Data .

\* In US\$

## 4 Conclusions

The Forest Harvest Planning Support (FHPS) model was created to attend the flexibility needs due to possible changes in the forest planning scenario. It can be applied to forest enterprises that use different harvest systems, or to those that destine timber to cellulose or charcoal production. This model may generate reduction in the costs during a competitive economical moment.

An increase of the cutting cost per hectare in areas with higher productivity levels does not exclude them from the optimum solution. This was verified because in sites with higher productivity the volume of timber cut per hour (machine's productivity) increased and consequently the cutting cost per stere of timber reduced.

The forest productivity levels recommended for cutting by the FHPS model are above 150 st/ha, while the parameters used to elaborate the harvest plan of the studied enterprise were in high cost levels. This recommends harvesting in forests with productivity above 100 st/ha, indicating that the adjustment model has the function of identifying new parameters in conditions in which empiric methods were used in the definition of the decision criteria. This facilitates the comprehension of the ideal productivity levels of the forest to be cut.

In this study's model the main decisive factor in the site selection was the productivity, having a direct impact in the cutting cost.

We identified the opportunity of testing the introduction of the exhaustion and charcoal making costs in the decision variables. Therefore, completing the productive cycle from the seedling formation until the transformation of timber in charcoal.

The adjustment cost always presented values equal to 0 or 1, characterizing a binary decision model.

The allocation simulations of the feller-buncher forest tractor in years 1 and 2 demonstrated the possibility of using the model in different time intervals. Because of this it is necessary to sub-divide the decision variables in smaller time intervals, therefore increasing the number of variables of the model according to the monitoring of smaller intervals necessity.

Nowadays the forest enterprises have been investing in modern information technology tools such as the Geographic Information System (GIS) and the Geographic Position System (GPS), for mapping and managing information. Therefore, the opportunity of constructing a single tool that aggregates the precision of the GPS and the versatility of the GIS as information source for the model developed in this study, could facilitate the interface between the mapping and planning areas. It could make the data income more agile, making it possible to be generated in only one base. This one tool could even bring more agility and precision to the forest planner's decisions and there are also gains inherent to the inevitable union of the inventory and planning areas.



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